DETERMINATION OF AVERAGE HEAT-TRANSFER COEFFICIENTS BY MEANS OF BUILT-IN α-CALORIMETERS

V. M. Kapinos, V. N. Pustovalov, and A. P. Rud'ko

UDC 536.2.083

The article describes an approximate method for determining the average heat-transfer coefficient on a surface upstream from a built-in α-calorimeter, on the basis of temperature measurements on the wall, as well as measurements of the local heat-transfer coefficient and the local flow velocity at the point of installation of the α-calorimeter.

Under industrial conditions, the measurement of heat-transfer coefficients by calorimetry of the entire heat-transferring or heat-receiving surface being investigated usually involves a great deal of difficulty. The tests may be considerably simplified by using built-in α-calorimeters. Depending on the design of the α-calorimeter, the heat-transfer coefficient is determined either by measuring the local heat flux [1-4] or indirectly from the nonstationary temperature field of the main body of the calorimeter [5, 6]. The heat-transfer coefficient measured by means of a thermal probe characterizes the intensity of heat exchange on the surface of the calorimeter itself [7]. This raises the question of the method to be used for the transformation from the measured heat-transfer coefficients to the actual values of the heat-transfer coefficients on the surface being investigated.

Let us first consider the case of a turbulent boundary layer flowing past a flat plate.

According to the approximate solution of [8], for an arbitrary law of variation of the velocity $U_0 = U_0(x)$ of the external flow and a temperature head $\Theta = \Theta(x)$, the local heat-transfer coefficient is given by the relation

$$\mathrm{Nu}_l = \frac{0.029\,\mathrm{Re}_l \mathrm{Pr}^{0.4}\Theta^{0.25}}{\left[\int\limits_0^x \frac{U_0}{\nu}\,\Theta^{1.25}dx\right]^{0,2}},\quad \mathrm{Nu}_l = \frac{\alpha_l x}{\lambda},\ \mathrm{Re}_l = \frac{U_0 x}{\nu}. \tag{1}$$

The solution (1) is based on the assumption that the law governing the heat exchange, in the form of a relationship between α_l and the number $\mathrm{Re}_t^{**} = U_0\delta_t^{**}/\nu$ is conservative with respect to the gradients of the velocity and the temperature head. For variable U_0 and increasing Θ, formula (1) has been experimentally confirmed many times.

The average heat-transfer coefficient is equal, by definition, to

$$\alpha = \frac{\int\limits_0^x \alpha_l \Theta dx}{\int\limits_0^x \Theta dx}. \tag{2}$$

Substituting into (2) the expression for α_l obtained from (1), we find

$$\mathrm{Nu} = 0.036\ \mathrm{Pr}^{0.4}\,\frac{x\left[\int\limits_0^x \frac{U_0}{\nu}\,\Theta^{1.25}dx\right]^{0.8}}{\int\limits_0^x \Theta dx}. \tag{3}$$

Lenin Polytechnic Institute, Khar'kov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 19, No. 2, pp. 236-242, August, 1970. Original article submitted July 23, 1969.

We assume that the measurements of the heat-transfer coefficients are made under isothermal conditions and that consequently $\Theta = 0$ everywhere except on the surface of the calorimeter. If we denote by x_0 and x_1 the coordinates of the leading and trailing edges of the calorimeter, then the distribution of the temperature head along the surface past which the flow takes place can be given in the following form: $\Theta = 0$ for $0 \leq x < x_0$ and $\Theta = \Theta_0$ for $x_0 \leq x \leq x_1$. Substituting Θ into (1) and (3) and setting $\Theta_0 = \text{const}$, we obtain for a flat plate ($U_0 = \text{const}$):

$$\mathrm{Nu}_l = 0{,}029\,\mathrm{Re}_l^{0{,}8}\,\mathrm{Pr}^{0{,}4}\left(\frac{x}{x - x_0}\right)^{0{,}2}, \tag{4}$$

$$\mathrm{Nu} = 0.036\ \mathrm{Re}^{0.8}\,\mathrm{Pr}^{0{,}4}\left(\frac{x_1}{x_1 - x_0}\right)^{0{,}2}, \quad x_0 \leqslant x \leqslant x_1. \tag{5}$$

The corresponding values of local and average heat-transfer coefficients will be measured by the α-calorimeter. Formulas (4) and (5) would seem to indicate that the measured heat-transfer coefficient depends on the position of the calorimeter with respect to the leading edge of the plate. However, we can easily convince ourselves that the coordinates x and x_1 can be eliminated from Eqs. (4) and (5). After some simplification, we obtain:

$$\mathrm{Nu}_l^0 = 0.029\,(\mathrm{Re}_l^0)^{0.8}\,\mathrm{Pr}^{0{,}4}, \quad \mathrm{Nu}_l^0 = \frac{\alpha_l\,(x - x_0)}{\lambda}, \quad \mathrm{Re}_l^0 = \frac{U_0(x - x_0)}{\nu};$$

$$\mathrm{Nu}^0 = 0{,}036\,(\mathrm{Re}^0)^{0.8}\,\mathrm{Pr}^{0{,}4}, \quad \mathrm{Nu}^0 = \frac{\alpha\,(x_1 - x_0)}{\lambda}, \quad \mathrm{Re}^0 = \frac{U_0\,(x_1 - x_0)}{\nu}.$$

From this it can be seen that irrespective of the actual distance from the leading edge of the plate, the α-calorimeter will measure the same heat-transfer coefficient value as if its leading edge coincided with the leading edge of the plate (we assume uniform flow in the boundary layer). In other words, a more or less substantial change in the hydrodynamic boundary layer does not affect the heat exchange. This fact, confirmed experimentally for a plate [9, 10] and a channel [5, 6] with unheated initial segments, is the result of pronounced changes that take place in the boundary layer when heat exchange occurs and far outweigh any prior preparation of its structure.

Thus, for isothermal conditions of heat transfer and constant flow velocity, it is a fairly simple matter to determine the true values of the heat-transfer coefficient from the measured values.

Now let us consider the case in which the calorimeter is preceded by a heat-transfer surface with an arbitrary distribution of temperature head and velocity. Experimental conditions of this kind occur, for example, in the investigation of heat transfer in various cavities of steam turbines: in spaces between cylinders, in chambers with unregulated steam bleeding, etc.

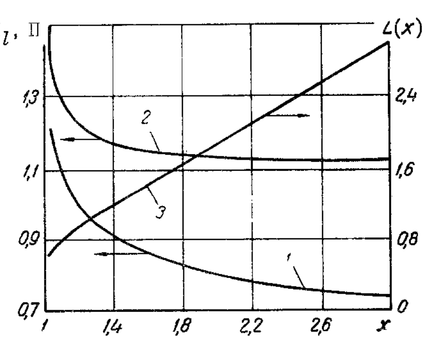
We shall assume that only the temperature distribution is measured along the entire heat-transferring surface but that at the cross section at which the α-calorimeter is installed we also make measurements of the local velocity (a Pitot tube is built into the calorimeter). From the measurement data we can calculate the integral

$$\int\limits_0^{x_0} \frac{U_0}{\nu}\,\Theta^{1.25}dx = \left[\frac{0.029\,\mathrm{Re}_l\mathrm{Pr}^{0.4}\,\Theta_0^{0.25}}{\mathrm{Nu}_l}\right]^5 - \frac{U_0}{\nu}\int\limits_{x_0}^{x^0} \Theta_0^{1.25}\,dx. \tag{6}$$

The expression (6) is obtained from (1) if the integral in (1) is represented as the sum of the respective integrals for the segment $0 \leq x \leq x_0$ before the calorimeter, where the distribution of the velocity U_0 is unknown, and the segment from the leading edge of the calorimeter to the cross section x^0 at which the local heat-transfer coefficient is being determined (if the calorimeter measures the average heat-transfer coefficient, then, in view of its small dimensions, the average value of α can be considered identical with its local value at the cross section $x^0 = 0.5\,(x_0 + x_1)$).

By substituting the integral (6) into (3), we can determine the average heat-transfer coefficient for the segment before the calorimeter by using only the temperature measurements on the surface past which the flow takes place.

Physically, the essence of the proposed method for determining the average heat-transfer coefficients from their local values as measured by a calorimeter lies in the fact that an α-calorimeter situated downstream with respect to the surface being investigated is conditioned by the prior hydrodynamic and thermal



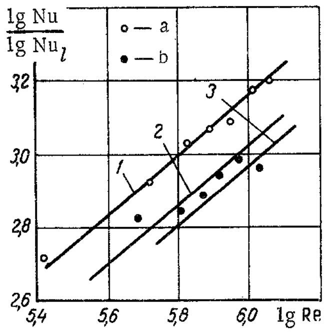


Fig. 1 Fig. 2

Fig. 1. Graphs of the functions Π, Π_l, L (curves 1, 2, 3, respectively).

Fig. 2. Comparison of experimental data with theoretical relations: 1) Nu_l according to (16); 2) Nu according to (15); 3) Nu according to (15) and (12); a) experimental values of the local Nusselt numbers Nu_l; b) Nu.

history of the flow. If there is no heat transfer on the surface upstream from the α-calorimeter, i.e., if $\Theta = 0$, and in addition $U_0 = U_0(x)$, then it is impossible to determine the heat-transfer coefficient on this surface by using an α-calorimeter.

It is natural to assume that the heat-transfer intensity measured by the calorimeter will not be greatly affected by any previous thermal perturbation of the flow. Quantitatively, this is manifested in the fact that the relative error in the determination of the average heat-transfer coefficient according to formulas (6) and (3) will be approximately 4-5 times the relative error in the measurement of the local heat-transfer coefficient by the built-in α-calorimeter. For this reason, the experimental data should not be processed on the basis of individual values of Nu_l; instead, the α-calorimeter measurement data should first be averaged in the form of an equation $Nu_l = cRe_l^n$, and then this relation should be "mapped" into a relation for the average Nusselt number Nu by making use of Eqs. (6) and (3).

It is obvious that the proposed method of determining the average heat transfer can be applied not only to a plate but also to other systems. The relation corresponding to Eq. (6) for the case of a plate and connecting the characteristics of the prior hydrodynamic and thermal history of the flow with the heat transfer at the α-calorimeter will be different for different cases. We shall describe below an experimental method for verifying the above procedure on a test apparatus designed for investigating heat transfer in the case of a freely rotating disk.

The measuring part of the apparatus consisted of a fiberglass laminate disk 600 mm in diameter, to the web of which, starting from a 200 mm diameter hub, annular electrical heaters made of foil 0.1 mm thick were attached. Each foil ring was 18 mm wide. The annular heaters were separated by spaces 2 mm wide and covered with epoxy resin. The heating rings of equal radius on the top and bottom of the disk were connected to each other in series, and the power of each pair of rings was independently regulated. The temperature of the foil was measured by means of 25 Chromel–Copel thermocouples, 5 of which were duplicates.

Relations analogous to (1) and (3) for a freely rotating disk with a hub can be obtained by generalizing Karman's solution for a disk with flow past it starting from its center. In [11] Karman tried to find an expression of the form $\delta = \text{const}\ r^{3/5}$ for the thickness of the turbulent boundary layer on the disk. If we take $\delta = r_0 y(\xi)(\nu/\omega r^2)^{1/5}$ and retain fractional-power profiles such as those in [11] for the radial and tangential components of the velocity, then in the case of a disk in a flow that begins at an intermediate radius r_0 the integral relations for the boundary layer yield the system of equations

$$\frac{d\Phi}{d\xi} = -3.6\frac{\Phi}{1+\xi} + 0.3306\left[\frac{(1+\xi)\varepsilon}{\Phi}\right]^{1/4}(1+\varepsilon^2)^{3/8}, \tag{7}$$

$$\frac{d\varepsilon}{d\xi} = \frac{\xi}{1+\xi} + \frac{0.1343}{\varepsilon(1+\xi)} - 0.4393\left(\frac{\varepsilon}{\Phi}\right)^{5/4}(1+\xi)^{1/4}(1+\varepsilon^2)^{3/8}, \tag{8}$$

where $\Phi(\xi) = \varepsilon(\xi)y(\xi)$; $\xi = (r-r_0)/r_0$. The tangential frictional stress can be expressed, in terms of the functions we have introduced, by the formula:

$$\frac{\tau\varphi}{\rho(\omega r)^2} = 0.0225\,\frac{T_0(x)}{\mathrm{Re}_l^{0.2}},\quad T_0(x) = = (1+\varepsilon^2)^{3/8}\left(\frac{x\varepsilon}{\Phi}\right)^{1/4},\quad x = \frac{r}{r_0}. \tag{9}$$

Accordingly, on the basis of Reynold's anology [12], the Nusselt number for a quadratic law governing the variation of the temperature head as a function of the disk radius will be equal (Pr = 1) to

$$\mathrm{Nu}_l = 0.0225\,\mathrm{Re}_l^{0.8}T_0(x),\ \mathrm{Nu}_l = \frac{\alpha_l r}{\lambda},\ \mathrm{Re}_l = \frac{\omega r^2}{\nu}. \tag{10}$$

The specific dependence (10) enables us to determine the numbers Nu_l and Nu for an arbitrary temperature head $\Theta(x)$ if we make the assumption, as we did in deriving Eq. (1), that the variation of heat transfer with respect to the radial temperature gradient obeys a conservative law. Omitting the calculations, which are given in detail for the case of a disk in [13-14], we give the final equations below:

$$\mathrm{Nu}_l = 0.0215\,\mathrm{Re}_l^{0.8}\mathrm{Pr}^{0.6}\frac{\Theta^{0.25}T_0J_0^{0.25}}{x^{0.5}\left[\int_1^x \Theta^{1.25}T_0J_0^{0.25}x^{1.1}\,dx\right]^{0.2}}, \tag{11}$$

$$\mathrm{Nu} = 0.0269\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.6}\,\frac{\left[\int_1^x \Theta^{1.25}T_0J_0^{0.25}x^{1.1}\,dx\right]^{0.8}}{x^{0.6}\int_1^x \Theta x\,dx}, \tag{12}$$

$$J_0 = \int_1^x x^{3.6}\,T_0(x)\,dx.$$

From this it follows that for Θ = const we have

$$\mathrm{Nu}_l = 0.0215\,\mathrm{Re}_l^{0.8}\mathrm{Pr}^{0.6}\,\Pi_l(x), \tag{13}$$

$$\mathrm{Nu} = 0.0269\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.6}\,\Pi(x), \tag{14}$$

$$\Pi_l(x) = \frac{T_0J_0^{0.25}}{x^{0.5}\left[\int_1^x T_0J_0^{0.25}x^{1.1}\,dx\right]^{0.2}};$$

$$\Pi(x) = \frac{2\left[\int_1^x T_0J_0^{0.25}x^{1.1}\,dx\right]^{0.8}}{x^{0.6}(x^2-1)}.$$

The functions $L = T_0J_0^{0.25}$, $\Pi_l(x)$, and $\Pi(x)$ were calculated on electronic computers (Fig. 1).

In the case under consideration, the desired integral, calculated for the heat-transfer surface upstream from the α-calorimeter, will be

$$\int_1^{x_1} \Theta^{1.25}T_0J_0^{0.25}x^{1.1}\,dx = \left[\frac{0.0215\,\mathrm{Re}_l^{0.8}\mathrm{Pr}^{0.6}\,\Theta^{0.25}T_0J^{0.25}}{\mathrm{Nu}_l x^{0.5}}\right]^5 - \int_{x_1}^{x_2} \Theta^{1.25}T_0J_0^{0.25}x^{1.1}\,dx. \tag{15}$$

By substituting this integral into Eq. (12), we can determine the average value of the heat-transfer coefficient on the annular surface $1 \le x \le x_1$.

The tests were conducted for seven values of the Reynolds number Re_l, ranging from $2.6 \cdot 10^5$ to $1.2 \cdot 10^6$. The excess temperature of the outermost pair of heating rings was kept at 40°C; on the rest of the web of the disk, from the hub radius r_0 = 100 mm to r_1 = 261 mm, the excess temperature was 20°C. The average heat-transfer coefficient on this part of the disk was determined by calculations made on the basis of the local coefficient measured by the α-calorimeter, and also, for comparison purposes, directly on the basis of the electrical power of the heating rings. As our α-calorimeter, we used the heating rings bounded by the 261 mm and 279 mm radii. The outermost heating rings served as a thermal barrier for the α-calorimeter.

In determining the local and average heat-transfer coefficients, we took into account the heat flow through the body of the disk between heating rings having different temperatures. The heat flux in the disk was found by simulation of the temperature field on the ÉGDA 9/60 electronic analog computer.

Curve 1 of Fig. 2, which approximates the experimental values of the local Nusselt numbers Nu_l, is described by the equation

$$Nu_l = 0.023\, Re_l^{0.8}. \tag{16}$$

in which the local Nusselt numbers Nu_l are 2% higher than the calculated values obtained from formula (11) for a stepwise distribution of temperature over the surface.

Curve 3 is the "mapping" of curve 1 in the sense indicated above, using Eqs. (15) and (12). For comparison purposes, we have also plotted on Fig. 2 another curve (curve 2) based on Eq. (12). The solid black circles represent the experimental values of the average Nusselt numbers. Since the determination of the average heat-transfer coefficient on the surface upstream from the α-calorimeter from the local values of the heat-transfer coefficient is such a complex problem, the agreement between the calculated and directly measured values of the heat-transfer coefficient should be considered satisfactory.

NOTATION

$U_0(x)$	is the flow velocity of the boundary layer at position x (along the surface);
Θ	is the excess temperature of the heat-transfer surface;
ν	is the kinematic viscosity;
Nu	is the Nusselt number;
Re	is the Reynolds number;
Pr	is the Prandtl number;
λ	is the thermal conductivity;
α	is the heat-transfer coefficient;
x_0, x_1	are the abscissae of the beginning and the end of the heated element of the α-calorimeter, respectively;
r_0	is the radius at which the flow past the disk begins;
r	is the local radius of the disk;
$x = r/r_0$	is the dimensionless value of the local disk radius;
δ	is the thickness of the hydrodynamic boundary layer;
ω	is the angular velocity of disk rotation;
Φ, y, T_0, J_0, ε	are the auxiliary functions;
δ_t^{**}	is the effective thermal boundary layer.

Subscripts

l	denotes local values.

LITERATURE CITED

1. P. K. Konakov et al., Heat Transfer in the Combustion Chambers of Steam Boilers [in Russian], Rechnoi Transport (1960).
2. A. M. Gurevich and V. V. Mitor, Énergomashinostroenie, No. 2 (1957).
3. S. I. Kosterin et al., Inzh.-Fiz. Zh., No. 4 (1962).
4. É. G. Narezhnyi, Sudostroenie, No. 10 (1957).
5. V. M. Kapinos and N. I. Nikitenko, International Journal of Heat and Mass Transfer, 6 (1963).
6. V. M. Kapinos and V. N. Pustovalov, Power Machinery Design. Interdepartmental Scientific and Technical Collection of the Ukrainian SSR [in Russian], Izd. Khar'kovskogo Gosuniversiteta, No. 1 (1965).
7. V. M. Kapinos and N. I. Nikitenko, Teploénergetika, No. 8 (1963).
8. G. S. Ambrok, Zh. Tekh. Fiz., 27, 9 (1957).
9. M. A. Mikheev, in: Convective and Radiative Heat Transfer [in Russian], Izd. AN SSSR (1960).
10. A. I. Chaplina, Inzh.-Fiz. Zh., No. 3 (1962).

11. T. H. Karman, ZAMM, 1 (1921).
12. L. A. Dorfman, Hydrodynamic Resistance and Heat Transfer in Rotating Bodies [in Russian], Fizmatgiz (1960).
13. V. M. Kapinos, Inzh.-Fiz. Zh., No. 1 (1964).
14. V. M. Kapinos, Izv. Akad. Nauk SSSR, Énergetika i Transport, No. 4 (1964).